

GRADE 11 STUDY GUIDE | 2024

Analytical Geometry

Mathematics

1. Distance, Midpoint, Gradient

$$\text{Distance: } d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\text{Midpoint: } M = ((x_1 + x_2)/2 ; (y_1 + y_2)/2)$$

$$\text{Gradient: } m = (y_2 - y_1)/(x_2 - x_1)$$

- Collinear points: gradient between any two pairs of the three points must be equal
- Parallel lines: equal gradients ($m_1 = m_2$)
- Perpendicular lines: $m_1 \times m_2 = -1$ (gradients are negative reciprocals)

Example: Points A(1;3), B(5;7), C(3;5) -- are they collinear?

$$m_{AB} = (7-3)/(5-1) = 4/4 = 1$$

$$m_{BC} = (5-7)/(3-5) = -2/-2 = 1$$

$m_{AB} = m_{BC}$ and B is common $\Rightarrow$ A, B, C are collinear.

2. Equation of a Line

$$\text{Slope-intercept form: } y = mx + c$$

$$\text{Point-slope form: } y - y_1 = m(x - x_1)$$

$$\text{General form: } ax + by + c = 0$$

To find the equation: need a gradient AND a point. Either given directly, or calculated from two given points.

3. Circles

$$\text{Standard form: } (x-a)^2 + (y-b)^2 = r^2$$

$$\text{Centre} = (a; b) \text{ and radius} = r$$

$$\text{General form: } x^2 + y^2 + Dx + Ey + F = 0 \Rightarrow \text{complete the square to convert}$$

Example: $x^2 + y^2 - 4x + 6y - 3 = 0$

$$\text{Complete square: } (x^2 - 4x + 4) + (y^2 + 6y + 9) = 3 + 4 + 9$$

$$\Rightarrow (x-2)^2 + (y+3)^2 = 16$$

Centre = (2; -3), radius = 4

Tangent to a Circle

A tangent is perpendicular to the radius at the point of tangency.

- Step 1: Find the gradient of the radius (from centre to the given point)
- Step 2: Tangent gradient = $-1/(\text{radius gradient})$
- Step 3: Use point-slope form with the point of tangency

4. Inclination Angle

$\tan(\theta) = m$, where θ is the angle with the positive x-axis

$\theta = \arctan(m)$, where $0 \leq \theta < 180^\circ$

If $m > 0$: θ is acute. If $m < 0$: θ is obtuse (between 90° and 180°).

Example: Find inclination of line $y = -2x + 3$

$m = -2$. $\tan(\theta) = -2$. $\theta = \arctan(-2) + 180 = 180 - 63.43 = 116.57^\circ$