GRADE 11 STUDY GUIDE | 2024

Trigonometry (Identities & Equations)

Mathematics

1. Trigonometric Ratios -- Review

sin(theta) = opp/hyp | cos(theta) = adj/hyp | tan(theta) = opp/adj = sin/cos

For any point (x; y) on the terminal arm of angle theta, with r = sqrt(x^2+y^2):

sin(theta) = y/r | cos(theta) = x/r | tan(theta) = y/x

KEY: CAST Rule -- which ratios are positive in each quadrant:
Q1: All positive | Q2: Sin | Q3: Tan | Q4: Cos

2. Special Angles

You must know these WITHOUT a calculator:

- 30 deg: sin=1/2, cos=sqrt(3)/2, tan=1/sqrt(3)
- 45 deg: sin=cos=1/sqrt(2), tan=1
- 60 deg: sin=sqrt(3)/2, cos=1/2, tan=sqrt(3)
- 0 deg: sin=0, cos=1, tan=0
- 90 deg: sin=1, cos=0, tan=undefined

3. Reduction Formulae (Quadrant Rules)

sin(180 - x) = sin(x)	**cos(180 - x) = -cos(x)**
sin(180 + x) = -sin(x)	**cos(180 + x) = -cos(x)**
sin(360 - x) = -sin(x)	**cos(360 - x) = cos(x)**
sin(-x) = -sin(x)	**cos(-x) = cos(x)**
sin(90 - x) = cos(x)	**cos(90 - x) = sin(x)**

Example: Simplify sin(150 deg)

sin(150) = sin(180 - 30) = sin(30) = 1/2

4. Trigonometric Identities

QUOTIENT: tan(x) = sin(x)/cos(x)

SQUARE: sin^2(x) + cos^2(x) = 1

=> sin^2(x) = 1 - cos^2(x)

=> cos^2(x) = 1 - sin^2(x)

Strategy for proving identities: work on ONE side only (usually the more complex side). Aim to transform it into the other side. Use substitution, factoring, and the identities above.

Example: Prove: (1 - cos^2(x))/sin(x) = sin(x)

LHS = sin^2(x)/sin(x) [using sin^2+cos^2=1]

= sin(x) = RHS checkmark

5. Trigonometric Equations

General method:

1. Isolate the trig ratio
2. Find the reference angle (always positive, in Q1)
3. Use CAST to find angles in the correct quadrants
4. Apply co-terminal angles by adding 360 deg x k (or 180 deg x k for tan)

If sin(x) = k: x = arcsin(k) + 360k OR x = 180 - arcsin(k) + 360k

If cos(x) = k: x = +/-arccos(k) + 360k

If tan(x) = k: x = arctan(k) + 180k

Example: Solve for x in [0; 360]: 2sin(x) - 1 = 0

sin(x) = 1/2

Ref angle = 30 deg

Q1: x = 30 deg | Q2: x = 180 - 30 = 150 deg

Answer: x = 30 deg or x = 150 deg

6. Graphs of Trig Functions

For y = a.sin(bx + p) + q (and similar for cos, tan):

- Amplitude = |a| (max distance from midline)
- Period = 360/b (for sin/cos) or 180/b (for tan)
- Phase shift = -p/b (horizontal shift)
- Vertical shift = q (midline: y = q)

Example: y = 2sin(x - 30) + 1

a=2: amplitude=2, b=1: period=360

Phase shift: -(-30)/1 = +30 (shifted 30 deg RIGHT)

Vertical shift: q=1 (midline y=1)

Max = 1+2 = 3, Min = 1-2 = -1

7. 2D Applications (Sine, Cosine, Area Rules)

Area of triangle = (1/2)ab.sin(C)

Sine Rule: a/sin(A) = b/sin(B) = c/sin(C)

Cosine Rule: a^2 = b^2 + c^2 - 2bc.cos(A)

KEY: Use Sine Rule when you know: AAS or ASA (angle-angle-side). Use Cosine Rule when you know: SAS or SSS.