

Grade 12 Mathematics: Trigonometry Study Guide

Grade 12 | Mathematics | Study Guide | 2024

Free Study Resources for South African Matric Learners

1. Trigonometric Ratios & Identities

$$\begin{aligned}\sin^2\theta + \cos^2\theta &= 1 \\ \tan\theta &= \sin\theta / \cos\theta \\ 1 + \tan^2\theta &= \sec^2\theta \\ 1 + \cot^2\theta &= \csc^2\theta\end{aligned}$$

2. Compound Angle Formulae

$$\begin{aligned}\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\ \cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\ \tan(A+B) &= (\tan A + \tan B) / (1 - \tan A \tan B)\end{aligned}$$

3. Double Angle Formulae

$$\begin{aligned}\sin 2A &= 2 \sin A \cos A \\ \cos 2A &= \cos^2 A - \sin^2 A \\ \cos 2A &= 2\cos^2 A - 1 \\ \cos 2A &= 1 - 2\sin^2 A \\ \tan 2A &= 2\tan A / (1 - \tan^2 A)\end{aligned}$$

4. Solving Trigonometric Equations

General solutions (θ in degrees):

$$\begin{aligned}\sin\theta = k: & \quad \theta = \arcsin(k) + 360^\circ \text{ OR } \theta = 180^\circ - \arcsin(k) + 360^\circ \\ \cos\theta = k: & \quad \theta = \pm \arccos(k) + 360^\circ \\ \tan\theta = k: & \quad \theta = \arctan(k) + 180^\circ\end{aligned}$$

TIP Always check your interval! If asked for θ in $[0^\circ; 360^\circ]$, substitute $n=0$ and $n=1$ to find all solutions.

5. Sine, Cosine & Area Rules

$$\begin{aligned}\text{Sine rule:} & \quad a/\sin A = b/\sin B = c/\sin C \\ \text{Cosine rule:} & \quad a^2 = b^2 + c^2 - 2bc \cos A \\ \text{Area of triangle:} & \quad \text{Area} = \frac{1}{2}ab \sin C\end{aligned}$$

When to use each rule

- Sine rule: given 2 angles + 1 side, OR 2 sides + non-included angle (watch for ambiguous case)
- Cosine rule: given 3 sides (find angle) OR 2 sides + included angle (find third side)
- Area rule: given 2 sides and the included angle

6. Trigonometric Graphs

$$y = a \sin(bx + p) + q$$

- $|a|$ = amplitude (half the vertical range)
- Period = $360 \text{ deg}/|b|$
- Phase shift = $-p/b$ (positive $\rightarrow$ left shift)
- Vertical shift = q (moves the midline up/down)

7. 2D & 3D Trigonometry

For 3D problems:

- 1) Identify the triangles in the diagram
- 2) Find unknown sides using sine/cosine rules in 2D triangles
- 3) Work step-by-step ? label all angles and sides
- 4) Angles of elevation/depression are measured from the horizontal