

GRADE 11 STUDY GUIDE | 2025

Trigonometry (Identities & Equations)

Mathematics

1. Trigonometric Ratios -- Review

$$\sin(\theta) = \text{opp/hyp} \quad | \quad \cos(\theta) = \text{adj/hyp} \quad | \quad \tan(\theta) = \text{opp/adj} = \sin/\cos$$

For any point (x; y) on the terminal arm of angle θ , with $r = \sqrt{x^2 + y^2}$:

$$\sin(\theta) = y/r \quad | \quad \cos(\theta) = x/r \quad | \quad \tan(\theta) = y/x$$

KEY: CAST Rule -- which ratios are positive in each quadrant:

Q1: All positive | Q2: Sin | Q3: Tan | Q4: Cos

2. Special Angles

You must know these WITHOUT a calculator:

- 30 deg: $\sin=1/2$, $\cos=\sqrt{3}/2$, $\tan=1/\sqrt{3}$
- 45 deg: $\sin=\cos=1/\sqrt{2}$, $\tan=1$
- 60 deg: $\sin=\sqrt{3}/2$, $\cos=1/2$, $\tan=\sqrt{3}$
- 0 deg: $\sin=0$, $\cos=1$, $\tan=0$
- 90 deg: $\sin=1$, $\cos=0$, $\tan=\text{undefined}$

3. Reduction Formulae (Quadrant Rules)

$$\sin(180 - x) = \sin(x) \quad \cos(180 - x) = -\cos(x)$$

$$\sin(180 + x) = -\sin(x) \quad \cos(180 + x) = -\cos(x)$$

$$\sin(360 - x) = -\sin(x) \quad \cos(360 - x) = \cos(x)$$

$$\sin(-x) = -\sin(x) \quad \cos(-x) = \cos(x)$$

$$\sin(90 - x) = \cos(x) \quad \cos(90 - x) = \sin(x)$$

Example: Simplify $\sin(150 \text{ deg})$

$$\sin(150) = \sin(180 - 30) = \sin(30) = 1/2$$

4. Trigonometric Identities

$$\text{QUOTIENT: } \tan(x) = \sin(x)/\cos(x)$$

$$\text{SQUARE: } \sin^2(x) + \cos^2(x) = 1$$

$$\Rightarrow \sin^2(x) = 1 - \cos^2(x)$$

$$\Rightarrow \cos^2(x) = 1 - \sin^2(x)$$

Strategy for proving identities: work on ONE side only (usually the more complex side). Aim to transform it into the other side. Use substitution, factoring, and the identities above.

Example: Prove: $(1 - \cos^2(x))/\sin(x) = \sin(x)$

$$\text{LHS} = \sin^2(x)/\sin(x) \quad [\text{using } \sin^2 + \cos^2 = 1]$$

$$= \sin(x) = \text{RHS} \quad \text{checkmark}$$

5. Trigonometric Equations

General method:

1. Isolate the trig ratio
2. Find the reference angle (always positive, in Q1)
3. Use CAST to find angles in the correct quadrants
4. Apply co-terminal angles by adding $360^\circ \times k$ (or $180^\circ \times k$ for tan)

$$\text{If } \sin(x) = k: x = \arcsin(k) + 360k \quad \text{OR} \quad x = 180 - \arcsin(k) + 360k$$

$$\text{If } \cos(x) = k: x = \pm \arccos(k) + 360k$$

$$\text{If } \tan(x) = k: x = \arctan(k) + 180k$$

Example: Solve for x in $[0; 360]$: $2\sin(x) - 1 = 0$

$$\sin(x) = 1/2$$

$$\text{Ref angle} = 30^\circ$$

$$\text{Q1: } x = 30^\circ \mid \text{Q2: } x = 180 - 30 = 150^\circ$$

$$\text{Answer: } x = 30^\circ \text{ or } x = 150^\circ$$

6. Graphs of Trig Functions

For $y = a \cdot \sin(bx + p) + q$ (and similar for cos, tan):

- Amplitude = $|a|$ (max distance from midline)
- Period = $360/b$ (for sin/cos) or $180/b$ (for tan)
- Phase shift = $-p/b$ (horizontal shift)
- Vertical shift = q (midline: $y = q$)

Example: $y = 2\sin(x - 30) + 1$

$$a=2: \text{amplitude}=2, b=1: \text{period}=360$$

$$\text{Phase shift: } -(-30)/1 = +30 \text{ (shifted } 30^\circ \text{ RIGHT)}$$

$$\text{Vertical shift: } q=1 \text{ (midline } y=1)$$

$$\text{Max} = 1+2 = 3, \text{ Min} = 1-2 = -1$$

7. 2D Applications (Sine, Cosine, Area Rules)

$$\text{Area of triangle} = (1/2)ab \cdot \sin(C)$$

$$\text{Sine Rule: } a/\sin(A) = b/\sin(B) = c/\sin(C)$$

$$\text{Cosine Rule: } a^2 = b^2 + c^2 - 2bc \cdot \cos(A)$$

KEY: Use Sine Rule when you know: AAS or ASA (angle-angle-side). Use Cosine Rule when you know: SAS or SSS.