

GRADE 11 STUDY GUIDE | 2025

Functions and Graphs

Mathematics

1. Introduction to Functions

A function is a rule that assigns exactly one output value to each input value. The set of all inputs is the DOMAIN and the set of all outputs is the RANGE.

KEY: Vertical Line Test: A graph represents a function if and only if every vertical line intersects the graph at most once.

- Notation: $f(x)$ reads "f of x" -- the value of function f at x
- Domain: all x -values for which $f(x)$ is defined
- Range: all corresponding y -values (outputs)
- One-to-one function: each x maps to a unique y AND each y comes from a unique x

2. Linear Functions $f(x) = mx + c$

The graph is a straight line. m = gradient (slope), c = y -intercept.

$$m = (y_2 - y_1) / (x_2 - x_1) \quad | \quad y = mx + c \quad | \quad \text{Parallel: } m_1 = m_2 \quad | \quad \text{Perpendicular: } m_1 \times m_2 = -1$$

Finding the equation of a straight line

- Given gradient and y -intercept: substitute directly into $y = mx + c$
- Given two points: first calculate m , then substitute one point to find c
- Given gradient and one point: substitute into $y - y_1 = m(x - x_1)$

Example: Line through (2; 5) and (-1; -4)

Step 1: $m = (-4 - 5) / (-1 - 2) = -9 / -3 = 3$

Step 2: $y = 3x + c$. Substitute (2;5): $5 = 6 + c$, so $c = -1$

Answer: $y = 3x - 1$

3. Quadratic Functions $f(x) = ax^2 + bx + c$

The graph is a PARABOLA. When $a > 0$ it opens upward (smile); when $a < 0$ it opens downward (frown).

$$\text{Turning point: } x = -b/(2a), \quad y = f(-b/(2a))$$

$$\text{Axis of symmetry: } x = -b/(2a)$$

$$\text{x-intercepts: solve } ax^2 + bx + c = 0$$

- Discriminant: $D = b^2 - 4ac$. If $D > 0$: two real roots; $D = 0$: one root; $D < 0$: no real roots



- If $a > 0$: range = [minimum value; $+\infty$)
- If $a < 0$: range = $(-\infty$; maximum value]
- y-intercept: set $x = 0$, giving $y = c$

Example: $f(x) = -x^2 + 4x + 5$

$a = -1$, $b = 4$, $c = 5$. TP: $x = -4/(2 \cdot -1) = 2$, $y = -4 + 8 + 5 = 9$. TP = (2;9)

Range: $y \leq 9$, i.e. $(-\infty$; 9]

x-intercepts: $-x^2 + 4x + 5 = 0 \Rightarrow x^2 - 4x - 5 = 0 \Rightarrow (x-5)(x+1) = 0 \Rightarrow x = 5$ or $x = -1$

4. Hyperbolic Functions $f(x) = a/(x-p) + q$

The graph has two branches and two asymptotes. The shape is called a hyperbola.

Vertical asymptote: $x = p$

Horizontal asymptote: $y = q$

Domain: $x \neq p$

Range: $y \neq q$

- x-intercept: set $y = 0$ and solve for x
- y-intercept: substitute $x = 0$
- If $a > 0$: quadrants 1 and 3 (relative to the point of intersection of asymptotes)
- If $a < 0$: quadrants 2 and 4

Example: $h(x) = 2/(x-1) + 3$

Asymptotes: $x = 1$ (vertical), $y = 3$ (horizontal)

x-intercept: $0 = 2/(x-1) + 3 \Rightarrow 2/(x-1) = -3 \Rightarrow x-1 = -2/3 \Rightarrow x = 1/3$

y-intercept: $h(0) = 2/(-1) + 3 = 1$, so (0;1)

5. Exponential Functions $f(x) = a \cdot b^x + q$, $b > 0$, $b \neq 1$

If $b > 1$: increasing (growth) | If $0 < b < 1$: decreasing (decay)

Horizontal asymptote: $y = q$

Domain: all real numbers

Range: $y > q$ (if $a > 0$) or $y < q$ (if $a < 0$)

Example: $g(x) = 3^x - 2$

Asymptote: $y = -2$. Domain: all reals. Range: $y > -2$

y-intercept: $g(0) = 1 - 2 = -1$, so (0;-1)

x-intercept: $0 = 3^x - 2 \Rightarrow 3^x = 2 \Rightarrow x = \log_3(2)$ approx 0.63

6. Transformations

Given $f(x)$, the following transformations apply:

- $f(x) + d$: shifts the graph d units UP ($d > 0$) or DOWN ($d < 0$)
- $f(x+c)$: shifts the graph c units LEFT ($c > 0$) or RIGHT ($c < 0$)
- $-f(x)$: reflects about the x-axis
- $f(-x)$: reflects about the y-axis
- $a \cdot f(x)$: vertically stretches ($|a| > 1$) or compresses ($|a| < 1$) the graph

7. Inverse Functions

The inverse of a function is found by swapping x and y , then making y the subject. The graph of f^{-1} is the reflection of f about the line $y = x$.

KEY: For f^{-1} to be a function, f must be one-to-one. If not, restrict the domain of f .

- Inverse of $f(x) = mx + c$: swap x and $y \Rightarrow x = my + c \Rightarrow y = (x-c)/m$
- Inverse of $f(x) = x^2$: swap to get $x = y^2 \Rightarrow y = \pm\sqrt{x}$. Restrict to $y \geq 0$ for a function.
- Inverse of $f(x) = a^x$: swap to get $x = a^y \Rightarrow y = \log_a(x)$