Grade 12 Mathematics: Algebra & Functions Study Guide

Grade 12 | Mathematics | Study Guide | 2024

Free Study Resources for South African Matric Learners

1. Algebraic Expressions & Equations

Factor Theorem

If f(a) = 0, then (x - a) is a factor of f(x). Use this to factorise cubic polynomials.

- Test x = +/-1, +/-2, +/-3, ... to find a root
- Once a factor is found, use long division or synthetic division
- Completely factorise the resulting quadratic

Remainder Theorem

When f(x) is divided by (x - a), the remainder is f(a).

2. Quadratic Equations & Inequalities

Quadratic formula:	`x = [-b +/- sqrt(b2-4ac)] / 2a`
Discriminant (Delta):	`Delta = b2 - 4ac`
Delta > 0:	`Two distinct real roots`
Delta = 0:	`One repeated real root (equal roots)`
Delta < 0:	`No real roots (complex roots)`

Quadratic Inequalities

To solve ax2 + bx + c > 0:

1) Find roots (where expression = 0)

2) Sketch parabola

3) Read off where graph is positive/negative

3. Exponential & Logarithmic Functions

Laws of Exponents

am . an	`= am?n`
am / an	`= am-n`
(am)n	`= amn`
a-n	`= 1/an`
a^(1/n)	`= nsqrta`
(ab)n	`= anbn`

Laws of Logarithms

log(ab)	`= log a + log b`
log(a/b)	`= log a - log b`
log(an)	`= n.log a`
log_a(x) = y	`<->  a? = x`
Change of base:	`log_a(x) = log(x)/log(a)`

4. Functions & Graphs

Types of Functions (Grade 12)

- Linear: f(x) = mx + c ? straight line, gradient m
- Quadratic: f(x) = ax2 + bx + c ? parabola
- Hyperbola: f(x) = a/(x+p) + q ? asymptotes at x=-p (vertical) and y=q (horizontal)
- Exponential: f(x) = a? + q ? asymptote at y=q
- Logarithmic: f(x) = log_a(x) ? inverse of exponential
- Trigonometric: sin, cos, tan with amplitude, period, phase shift

Inverse Functions

To find inverse: swap x and y, then solve for y.

Restriction: inverse must also be a function (use domain restriction if needed).

f and f-1 are reflections of each other in the line y = x.

5. Sequences & Series

Arithmetic Sequences

General term:	`Tn = a + (n-1)d`
Sum of n terms:	`Sn = n/2.(2a + (n-1)d)  or  n/2.(a + l)`
Common diff:	`d = T2 - T1 = T3 - T2`

Geometric Sequences

General term:	`Tn = a.rn-1`
Sum of n terms:	`Sn = a(rn - 1)/(r - 1)  for r != 1`
Infinite sum:	`Sinf = a/(1-r)  for |r| < 1`
Common ratio:	`r = T2/T1`

6. Financial Mathematics

Simple interest:	`A = P(1 + in)`
Compound interest:	`A = P(1 + i)n`
Effective rate:	`ieff = (1 + ino?/m)m - 1`
Future value annuity:	`F = x[(1+i)n - 1]/i`
Present value annuity:	`P = x[1-(1+i)-n]/i`

TIP Be sure to convert % to decimal (e.g. 8% = 0.08) and convert annual rate to period rate (e.g. monthly: i = annual rate / 12).