Grade 12 Mathematics: Calculus Study Guide

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Free Study Resources for South African Matric Learners

1. Introduction to Calculus

Calculus is divided into two main branches: Differential Calculus (finding derivatives) and Integral Calculus (finding anti-derivatives). In Grade 12, the focus is primarily on differentiation and its applications.

2. Limits

A limit describes the value a function approaches as the input approaches a certain value.

Definition

lim f(x) as x->a = L means f(x) gets arbitrarily close to L as x approaches a.

- lim (x2) as x->3 = 9
- lim (sin x / x) as x->0 = 1 (important result)
- If f(a) is defined, lim f(x) as x->a = f(a) (continuity)

3. Differentiation from First Principles

The derivative of f(x) from first principles is defined as:

f'(x) = `lim [f(x+h) - f(x)] / h  as h -> 0`

Example: Find f'(x) for f(x) = x2

f'(x) = lim [(x+h)2 - x2] / h = lim [x2 + 2xh + h2 - x2] / h = lim [2x + h] = 2x

TIP Matric exams often ask you to differentiate a specific function from first principles. Practice x2, x3, and 1/x.

4. Rules of Differentiation

Power rule:	`d/dx [xn] = n.xn-1`
Constant rule:	`d/dx [c] = 0`
Sum/diff rule:	`d/dx [f+/-g] = f'+/-g'`
Constant multiple:	`d/dx [c.f(x)] = c.f'(x)`
Chain rule:	`d/dx [f(g(x))] = f'(g(x)).g'(x)`

5. Standard Derivatives

d/dx [xn]	`= n.xn-1`
d/dx [sqrtx]	`= 1/(2sqrtx)`
d/dx [1/x]	`= -1/x2`
d/dx [sin x]	`= cos x`
d/dx [cos x]	`= -sin x`
d/dx [e?]	`= e?`
d/dx [ln x]	`= 1/x`

6. Applications of Differentiation

Tangent & Normal Lines

The gradient of the tangent to y=f(x) at x=a is f'(a). Equation of tangent: y - f(a) = f'(a)(x - a).

The normal is perpendicular: gradient of normal = -1/f'(a).

Stationary Points (Turning Points)

- Set f'(x) = 0 and solve for x
- Second derivative test: if f''(x) > 0 -> local minimum; f''(x) < 0 -> local maximum
- If f''(x) = 0, test first derivative sign change (concavity test)

Points of Inflection

Occur where f''(x) = 0 AND changes sign. These are where the curve changes concavity.

7. Cubic Functions & Sketching

For f(x) = ax3 + bx2 + cx + d:

- Find x-intercepts: set f(x) = 0 (use factor theorem)
- Find y-intercept: set x = 0 -> y = d
- Find turning points: set f'(x) = 0
- Find inflection point: set f''(x) = 0
- Determine end behaviour from sign of a (a>0 -> rises right; a<0 -> falls right)

8. Optimisation Problems

Optimisation uses calculus to find maximum or minimum values in real-world problems.

TIP Steps: 1) Write an equation for the quantity to maximise/minimise. 2) Express in terms of one variable. 3) Differentiate and set equal to 0. 4) Confirm it's a max or min using second derivative.

Example: A box with open top is made from a 30cm x 30cm sheet by cutting squares of side x from each corner. Maximise volume.

V = x(30-2x)2 -> V'= (30-2x)2 + x.2(30-2x)(-2) = (30-2x)(30-6x) -> x = 5 cm gives max volume.