

Grade 12 Mathematics: Analytical Geometry Study Guide

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Free Study Resources for South African Matric Learners

1. Key Formulae

Distance:	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Midpoint:	$M = ((x_1 + x_2)/2, (y_1 + y_2)/2)$
Gradient:	$m = (y_2 - y_1)/(x_2 - x_1)$
Line equation:	$y - y_1 = m(x - x_1)$
Parallel lines:	$m_1 = m_2$
Perpendicular:	$m_1 \times m_2 = -1$
Circle (centre O):	$x^2 + y^2 = r^2$
Circle (centre (a,b)):	$(x - a)^2 + (y - b)^2 = r^2$

2. Circles

Equation of a Circle

General form: $x^2 + y^2 + Dx + Ey + F = 0$

To find centre and radius: complete the square for x and y terms.

Example: $x^2 + y^2 - 6x + 4y - 12 = 0$

-> $(x^2 - 6x + 9) + (y^2 + 4y + 4) = 12 + 9 + 4$

-> $(x - 3)^2 + (y + 2)^2 = 25$

-> Centre (3, -2), radius = 5

Tangent to a Circle

At point (x_1, y_1) on $x^2 + y^2 = r^2$:

Tangent equation: $x \cdot x_1 + y \cdot y_1 = r^2$

The radius to the point is perpendicular to the tangent.

3. Inclination Angle

The inclination angle α of a line is the angle measured anti-clockwise from the positive x-axis.

$m = \tan \alpha$, so $\alpha = \arctan(m)$.

If $m < 0$, then α in (90 deg; 180 deg).

4. Coordinate Geometry Proofs

Proving Quadrilateral Properties

- Parallelogram: opposite sides equal AND parallel (same gradient)
- Rectangle: parallelogram with perpendicular adjacent sides ($m_1 \times m_2 = -1$)
- Rhombus: parallelogram with all sides equal
- Square: rectangle with all sides equal
- Trapezium: exactly one pair of parallel sides

TIP In exam proofs, show gradient calculations clearly. State "therefore the lines are parallel/perpendicular" explicitly.
Show distance formula fully.

5. Dividing a Line Segment

Point P divides line segment AB in ratio m:n from A:

$$\begin{aligned}x_P &= (n \cdot x_A + m \cdot x_B) / (m+n) \\y_P &= (n \cdot y_A + m \cdot y_B) / (m+n)\end{aligned}$$