

Grade 12 Mathematics: Algebra & Functions Study Guide

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Free Study Resources for South African Matric Learners

1. Algebraic Expressions & Equations

Factor Theorem

If $f(a) = 0$, then $(x - a)$ is a factor of $f(x)$. Use this to factorise cubic polynomials.

- Test $x = +/-1, +/-2, +/-3, \dots$ to find a root
- Once a factor is found, use long division or synthetic division
- Completely factorise the resulting quadratic

Remainder Theorem

When $f(x)$ is divided by $(x - a)$, the remainder is $f(a)$.

2. Quadratic Equations & Inequalities

Quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Discriminant (Delta): $\Delta = b^2 - 4ac$

Delta > 0: Two distinct real roots

Delta = 0: One repeated real root (equal roots)

Delta < 0: No real roots (complex roots)

Quadratic Inequalities

To solve $ax^2 + bx + c > 0$:

- 1) Find roots (where expression = 0)
- 2) Sketch parabola
- 3) Read off where graph is positive/negative

3. Exponential & Logarithmic Functions

Laws of Exponents

$a^m \cdot a^n = a^{m+n}$

$a^m / a^n = a^{m-n}$

$(a^m)^n = a^{mn}$

$a^{-n} = 1/a^n$

$a^{1/n} = \sqrt[n]{a}$

$(a^m)^n = a^{mn}$

Laws of Logarithms

$\log(ab) = \log a + \log b$

$\log(a/b) = \log a - \log b$

$\log(a^n) = n \cdot \log a$

$\log_a(x) = y \iff a^y = x$

Change of base: $\log_a(x) = \frac{\log(x)}{\log(a)}$

4. Functions & Graphs

Types of Functions (Grade 12)

- Linear: $f(x) = mx + c$? straight line, gradient m
- Quadratic: $f(x) = ax^2 + bx + c$? parabola
- Hyperbola: $f(x) = a/(x+p) + q$? asymptotes at $x=-p$ (vertical) and $y=q$ (horizontal)
- Exponential: $f(x) = a^x + q$? asymptote at $y=q$
- Logarithmic: $f(x) = \log_a(x)$? inverse of exponential
- Trigonometric: $\sin, \cos, \tan$ with amplitude, period, phase shift

Inverse Functions

To find inverse: swap x and y , then solve for y .

Restriction: inverse must also be a function (use domain restriction if needed).

f and f^{-1} are reflections of each other in the line $y = x$.

5. Sequences & Series

Arithmetic Sequences

General term: $T_n = a + (n-1)d$

Sum of n terms: $S_n = n/2 \cdot (2a + (n-1)d)$ or $n/2 \cdot (a + l)$

Common diff: $d = T_2 - T_1 = T_3 - T_2$

Geometric Sequences

General term: $T_n = a \cdot r^{n-1}$

Sum of n terms: $S_n = a(r^n - 1)/(r - 1)$ for $r \neq 1$

Infinite sum: $S_{\infty} = a/(1-r)$ for $|r| < 1$

Common ratio: $r = T_2/T_1$

6. Financial Mathematics

Simple interest: $A = P(1 + in)$

Compound interest: $A = P(1 + i)^n$

Effective rate: $i_{\text{eff}} = (1 + i/n)^n - 1$

Future value annuity: $F = x[(1+i)^n - 1]/i$

Present value annuity: $P = x[1 - (1+i)^{-n}]/i$

TIP Be sure to convert % to decimal (e.g. 8% = 0.08) and convert annual rate to period rate (e.g. monthly: $i = \text{annual rate} / 12$).