

Grade 12 Mathematics: Calculus Study Guide

Grade 12 | Mathematics | Study Guide | 2024

Free Study Resources for South African Matric Learners

1. Introduction to Calculus

Calculus is divided into two main branches: Differential Calculus (finding derivatives) and Integral Calculus (finding anti-derivatives). In Grade 12, the focus is primarily on differentiation and its applications.

2. Limits

A limit describes the value a function approaches as the input approaches a certain value.

Definition

$\lim_{x \rightarrow a} f(x) = L$ means $f(x)$ gets arbitrarily close to L as x approaches a .

- $\lim_{x \rightarrow 3} (x^2) = 9$
- $\lim_{x \rightarrow 0} (\sin x / x) = 1$ (important result)
- If $f(a)$ is defined, $\lim_{x \rightarrow a} f(x) = f(a)$ (continuity)

3. Differentiation from First Principles

The derivative of $f(x)$ from first principles is defined as:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Example: Find $f'(x)$ for $f(x) = x^2$

$$f'(x) = \lim_{h \rightarrow 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - x^2}{h} = \lim_{h \rightarrow 0} \frac{2xh + h^2}{h} = \lim_{h \rightarrow 0} (2x + h) = 2x$$

TIP Matric exams often ask you to differentiate a specific function from first principles. Practice x^2 , x^3 , and $1/x$.

4. Rules of Differentiation

Power rule:	$\frac{d}{dx} [x^n] = n \cdot x^{n-1}$
Constant rule:	$\frac{d}{dx} [c] = 0$
Sum/diff rule:	$\frac{d}{dx} [f \pm g] = f' \pm g'$
Constant multiple:	$\frac{d}{dx} [c \cdot f(x)] = c \cdot f'(x)$
Chain rule:	$\frac{d}{dx} [f(g(x))] = f'(g(x)) \cdot g'(x)$

5. Standard Derivatives

$\frac{d}{dx} [x^n]$	$= n \cdot x^{n-1}$
$\frac{d}{dx} [\sqrt{x}]$	$= \frac{1}{2\sqrt{x}}$
$\frac{d}{dx} [1/x]$	$= -1/x^2$
$\frac{d}{dx} [\sin x]$	$= \cos x$
$\frac{d}{dx} [\cos x]$	$= -\sin x$
$\frac{d}{dx} [e^x]$	$= e^x$
$\frac{d}{dx} [\ln x]$	$= 1/x$

6. Applications of Differentiation

Tangent & Normal Lines

The gradient of the tangent to $y=f(x)$ at $x=a$ is $f'(a)$. Equation of tangent: $y - f(a) = f'(a)(x - a)$.

The normal is perpendicular: gradient of normal = $-1/f'(a)$.

Stationary Points (Turning Points)

- Set $f'(x) = 0$ and solve for x
- Second derivative test: if $f''(x) > 0 \rightarrow$ local minimum; $f''(x) < 0 \rightarrow$ local maximum
- If $f''(x) = 0$, test first derivative sign change (concavity test)

Points of Inflection

Occur where $f''(x) = 0$ AND changes sign. These are where the curve changes concavity.

7. Cubic Functions & Sketching

For $f(x) = ax^3 + bx^2 + cx + d$:

- Find x-intercepts: set $f(x) = 0$ (use factor theorem)
- Find y-intercept: set $x = 0 \rightarrow y = d$
- Find turning points: set $f'(x) = 0$
- Find inflection point: set $f''(x) = 0$
- Determine end behaviour from sign of a ($a > 0 \rightarrow$ rises right; $a < 0 \rightarrow$ falls right)

8. Optimisation Problems

Optimisation uses calculus to find maximum or minimum values in real-world problems.

TIP Steps: 1) Write an equation for the quantity to maximise/minimise. 2) Express in terms of one variable. 3) Differentiate and set equal to 0. 4) Confirm it's a max or min using second derivative.

Example: A box with open top is made from a 30cm x 30cm sheet by cutting squares of side x from each corner. Maximise volume.

$$V = x(30-2x)^2 \rightarrow V' = (30-2x)^2 + x \cdot 2(30-2x)(-2) = (30-2x)(30-6x) \rightarrow x = 5 \text{ cm gives max volume.}$$